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A New Method for Measurement of Complex Permittivity of Liquids Using the Phase Information of Standing Waves

Hanbao Jiang, Mingyi Sun, and Wenjing Chen

***Abstract*— A new approach to determine the propagation constant, $\gamma=\alpha+j\beta$, of waves on a transmission line from phase measurements is proposed in this paper. This new method is very suitable for determining small α. Its distinctive feature is that the attenuation constant α of waves on the transmission line is the slope of a linear function of the displacement of a detector. Thus the attenuation constant α can be determined accurately even if it is very small.**

I. Introduction

Variable-length liquid sample cells have been widely used to measure the complex permittivity of liquids at microwave frequencies. Van Loon *et al.* [1] used the power reflected from a variable-length liquid cell, and Stumper [2] used the power transmitted through a variable-length inclined liquid column to obtain propagation constant. Buckmaster *et al.* [3] recently reported the measurement of the complex permittivity of high-loss liquids by measuring the phase constant and attenuation constant of traveling waves which penetrates a variable-length liquid column. We also made a swept-frequency measurement of the complex permittivity of saline water by using a slotted line [4], in which traveling waves were established.

Thus, except for high-loss liquid conditions in which traveling waves can easily be established, most of the previous works only rely on the amplitude information of standing waves, ignoring the phase information. This causes much difficulty when measuring small α, and requires complicated mathematical processing [5]. The phase variation of a standing wave is highly sensitive to α when it is small. The relation between α and the phase shift on a transmission line is relatively simple. Thus, when α is small, it can be determined more easily and accurately by using phase information than by using amplitude information of a standing wave.

In this paper, we will first discuss the phase characteristics of standing waves on a transmission line in general conditions. Based on the phase information of standing waves, a new method for measuring the complex permittivity of low-loss liquids will be proposed.

Preliminary experiments were performed to prove the new method, and the results show good agreement with the theoretical values and other experimental values. The method provides a new approach to determine the attenuation constant when it is small. The other features of the method include wide-band operation, simple mathematical calculation, and compatibility with the variable-length liquid sample cell method used now.

II. Theory

On a uniform transmission line, in general a condition, there are standing waves which can be described by

$$V(z\cdot l)=\frac{V_s Z_0}{Z_s+Z_0}\,\frac{e^{-\gamma z}+\rho_T e^{-2\gamma l}e^{\gamma z}}{1-\rho_T\rho_s e^{-2\gamma l}} \tag{1}$$

where ρ_T and ρ_s are the complex reflective coefficient of load and source, respectively, Z_o is the characteristic impedance of the transmission line, Z_s is the source impedance, V_s is the voltage of the source, and l is the length of the transmission line.

Two types of standing wave patterns will be used to determine the propagation constant of waves on a transmission line.

A. The Voltage Standing Wave Distribution Between Load End and Source End

In this condition, the amplitude and phase shift of standing waves are [6]

$$|V(d)|=\left|2V_1 e^{-\gamma l}\sqrt{\rho_T}\right|\left[\sinh^2(\alpha d+p)+\cos^2(\beta d+q)\right]^{1/2} \tag{2a}$$

$$\varphi(d)=\tan^{-1}\left[\tanh(\alpha d+p)\tan(\beta d+q)\right] \tag{2b}$$

where

$$V_1=\frac{V_s Z_0}{(Z_s+Z_0)(1-\rho_T\rho_s e^{-2\gamma l})}$$

d is the distance from load end to the point where the voltage is measured, $z+d=l$, and $p=\ln(|\rho_T|)^{-1/2}, q=-\frac{1}{2}\varphi_r$.

Equations (2a) and (2b) explicitly show that either the amplitude or the phase of a standing wave contains the information of the propagation constant and the load. Fig. 1 shows the phase distributions of standing waves along a transmission line. From Fig. 1, we can see that any phase distribution of waves on a transmission line lies between two lines: one is the straight line, representing a traveling wave; the other is the zigzag line, representing a pure standing wave.

Furthermore, from (2b) and Fig. 1, we can see that the distance D between the two sequential points where the standing wave phase passes through $\pi/2, 3\pi/2\cdots(2n+1)\pi/2$ is exactly equal to $\lambda/2$ of waves on the transmission line. So β is determined by $\beta=\pi/D$. Compared to using the amplitude information of standing waves, the distance between two minimums of the amplitude is not exactly equal to $\lambda/2$ unless the loss of the transmission line can be ignored [5]. From (2b), we can get another form of the equation:

$$L=\alpha d+p=\tanh^{-1}\left[\frac{\tan\varphi(d)}{\tan(\beta d+q)}\right] \tag{3a}$$

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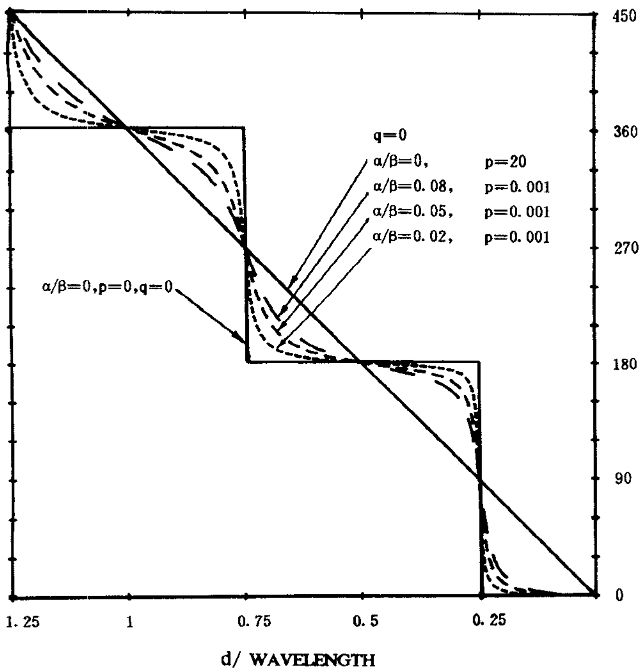

Fig. 1. The phase shift of standing waves on a transmission line having different values of α/β. The solid straight line represents the linear phase shift of traveling waves along the line, and the zigzag line represents the step changes of the phase shift of pure standing waves. All possible phase variations on a transmission line lie between these two lines.

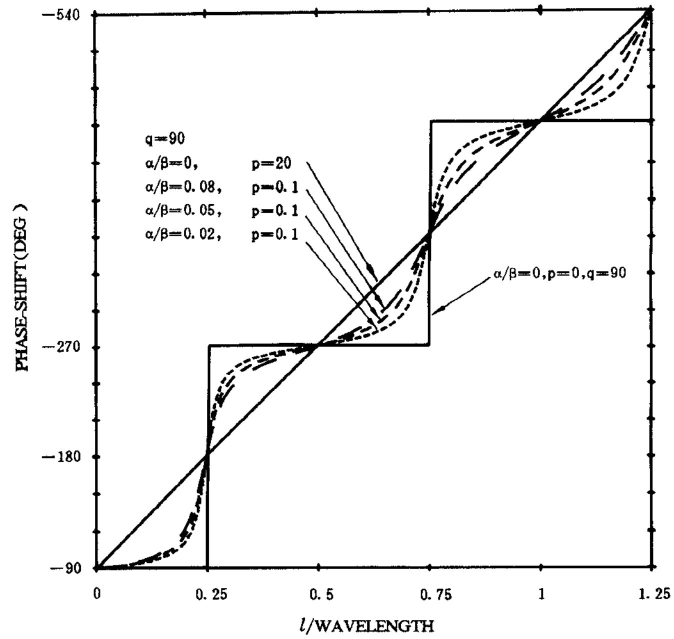

Fig. 2. The phase variations on a movable load end when it moves along transmission lines having different values α/β. All possible phase variations on the movable load lie between two lines. One of them represents phase linear decrease of traveling waves, the other represents phase step changes of pure standing waves.

$$\alpha = \frac{1}{d - d_o} \tanh^{-1} \left\{ \frac{\tan \varphi(d)}{\tan [\beta(d - d_0)]} \right\} \tag{3b}$$

where $L = \alpha d + p$ can be regarded as the total loss on the line at the distance d from the load end. So (3a) shows that the total loss L on a transmission line is a linear function with the distance from the load end. The slope of the linear function is just the attenuation constant α to be determined and shown clearly in (3b), where $d - d_o$ is an incremental displacement of a probe, and d_0 is any position where $\varphi(d_0) = 0$. So far, we have a new approach to determine the attenuation constant of waves on a transmission line from the phase measurement.

The variation rate of $\varphi(d)$ with α can be obtained from (2b)

$$\frac{\partial \varphi(d)}{\partial \alpha} = d \frac{\cos \varphi(d) \sin \varphi(d)}{\cosh(\alpha d + p) \sinh(\alpha d + p)}. \tag{4a}$$

When measuring low-loss liquids, (4a) can be simplified to

$$\frac{\partial \varphi(d)}{\partial \alpha} \doteq \frac{\cos \varphi(d) \sin \varphi(d)}{\alpha}. \tag{4b}$$

Therefore, (4b) shows that using phase information of standing waves is especially suitable for determining small α.

B. The Voltage Standing Wave Patterns on a Movable Load End

In this condition, (1) becomes

$$V(l) = V \frac{(1 + \rho_T) e^{-\gamma l}}{1 - \rho_T \rho_S e^{-2\gamma l}} \tag{5}$$

where $V = V_S Z_0 / (Z_S + Z_0)$. The phase shift of $V(l)$ on the movable load end is

$$\varphi(l) = -\tan^{-1} \left[\frac{\tan(\beta l + q)}{\tanh(\alpha l + p)} \right] \tag{6}$$

where $p = \ln(|\rho_T \rho_S|)^{-1/2}$, and $q = -\frac{1}{2}(\varphi_T + \varphi_S)$. Fig. 2 shows the phase variations on the load end when it moved along the line.

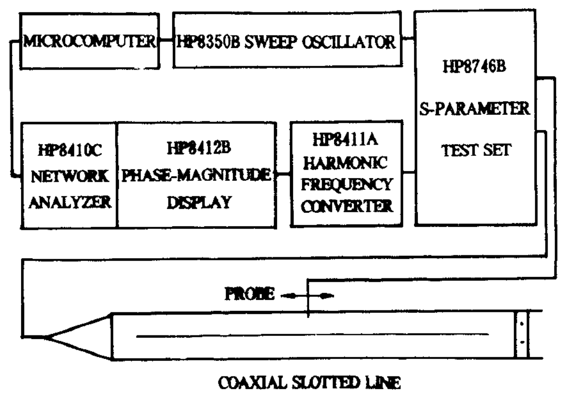

Fig. 3. The experimental setup for measuring the complex permittivity of liquids from the phase information of standing waves.

Also, from (6) and Fig. 2, the phase constant β can be determined accurately by measuring these positions where $\varphi(l)$ passes through $\pi, 2\pi, 3\pi, \cdots, n\pi$. Again, $\beta = \pi/D$, where D is the distance between two sequential positions mentioned above. In the same way as in (3), we get the following equation for determining the attenuation constant α:

$$L = \alpha l + p = \tanh^{-1} \left[\frac{\tan(\beta l + q)}{\tan \varphi(l)} \right] \tag{7a}$$

$$\alpha = \frac{1}{l - l_0} \tanh^{-1} \left\{ \frac{\tan[\beta(l - l_0)]}{\tan \varphi(l)} \right\}. \tag{7b}$$

Therefore, the total loss L at the movable load is also a linear function of the distance l. The slope of the linear function is the attenuation constant of waves on the line, $l - l_0$ is an incremental displacement of the movable load, and l_0 is any position where $\varphi(l_0) = 0$ or π.

TABLE I
THE COMPLEX PERMITTIVITY ϵ' AND ϵ'' OF DEIONIZED WATER DETERMINED FROM PHASE INFORMATION OF STANDING WAVES. THE TESTING FREQUENCY WAS 0.2 GHz

temperature (℃)	β (rad/m)	$\delta\beta$	α (Nep/m)	$\delta\alpha$	ϵ'	$\Delta\epsilon'$	ϵ''	$\Delta\epsilon''$	ref. ϵ'	value[*] ϵ''
15	37.93	0.063	0.23	0.01	81.97	0.54	1.00	0.09		
25	37.12	0.072	0.19	0.01	78.51	0.61	0.80	0.09	78.52	0.86
27	36.82	0.066	0.19	0.007	77.28	0.54	0.81	0.06		

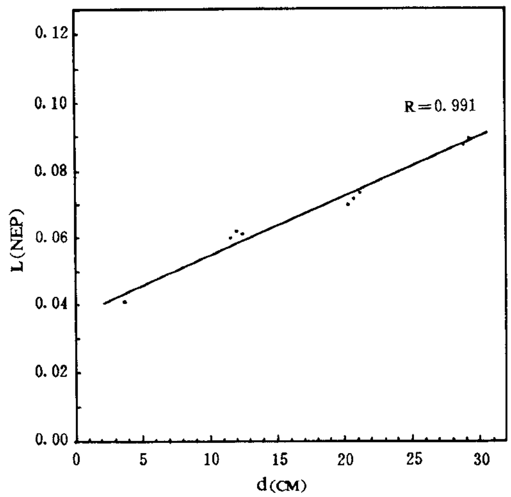

Fig. 4. Graph showing the total loss L as a linear function of displacement d of a probe for determining the attenuation constant

The variation rate of $\varphi(l)$ with α is the same as in (4). Again, we can determine the attenuation constant of waves from phase measurement.

III. EXPERIMENTAL PROCEDURE AND RESULTS

Preliminary experiments were performed based on the standing wave distributions between the load end and the source. The basic experimental system shown in Fig. 3 consisted of an HP8410C network analyzer system controlled by a microcomputer. A coaxial slotted line was filled with deionized water to be tested, and a nontuned, wide-band probe was used to detect both the amplitude and phase shift of standing waves established on the slotted line. First, the phase constant β is determined according to the distance D between two sequential positions where the phase of the standing wave passes through $\pi/2, 3\pi/2 \cdots (2n+1)\pi/2$. This can be seen on the screen of an HP8412B because at these points the phase changes abruptly just as in Fig. 1. Then a midpoint between two sequential positions determined previously is selected as the starting point where the phase is set to zero. Moving the probe from the starting position toward the other end of the slotted line, the phase shift of standing wave $\varphi(d)$ in as many as possible positions and in different quadrants is measured and stored in the computer. The attenuation constant α is calculated as the regression coefficient using linear regression processing. The correlation coefficient R is used as the criteria for the measurement, and $R > 0.95$ was selected as a goodness in our preliminary measurement. The frequency used was 0.2 GHz.

Fig. 4 shows the relation between the total loss L determined experimentally on the slotted line with the displacement d from the starting position. It clearly shows that the relation is very close to a straight line as predicted by (3a). The correlation coefficient R is 0.991 in the experiment. Table I lists the complex permittivity of deionized water tested by this new method at several temperatures. The $\delta\beta$ and $\delta\alpha$ in the table are the standard deviations of β and α, respectively, $\Delta\epsilon'$, $\Delta\epsilon''$ are the measurement uncertainties (two standard deviations) of ϵ' and ϵ'', respectively. The results show good agreement with cole-cole theory values [7] and the experimental values obtained by Stuchly and Kraszewski [8].

IV. SUMMARY

Using the phase information of standing waves, the propagation constant, $\gamma = \alpha + j\beta$, of waves on a transmission line can be determined more easily and accurately than by using only amplitude information of standing waves when α is small. The distinctive feature of this new method is that the attenuation constant α is the slope of a linear function with the displacement of a detector from an initial position. So even if α is very small, it can also be determined accurately.

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